OCR
RECOGNISING ACHIEVEMENT

ADVANCED GCE

MATHEMATICS

Core Mathematics 3

4723

Candidates answer on the Answer Booklet

OCR Supplied Materials:
- 8 page Answer Booklet
- List of Formulae (MF1)

Other Materials Required:
- Scientific or graphical calculator

Wednesday 9 June 2010
Afternoon

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- Write your name clearly in capital letters, your Centre Number and Candidate Number in the spaces provided on the Answer Booklet.
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully and make sure that you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- You are permitted to use a graphical calculator in this paper.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets **[ ]** at the end of each question or part question.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- This document consists of **4** pages. Any blank pages are indicated.

1 Find $\frac{dy}{dx}$ in each of the following cases:

(i) $y = x^3e^{2x}$, **[2]**

(ii) $y = \ln(3 + 2x^2)$, **[2]**

(iii) $y = \frac{x}{2x+1}$. **[2]**

2 The transformations R, S and T are defined as follows.

R : reflection in the x-axis
S : stretch in the x-direction with scale factor 3
T : translation in the positive x-direction by 4 units

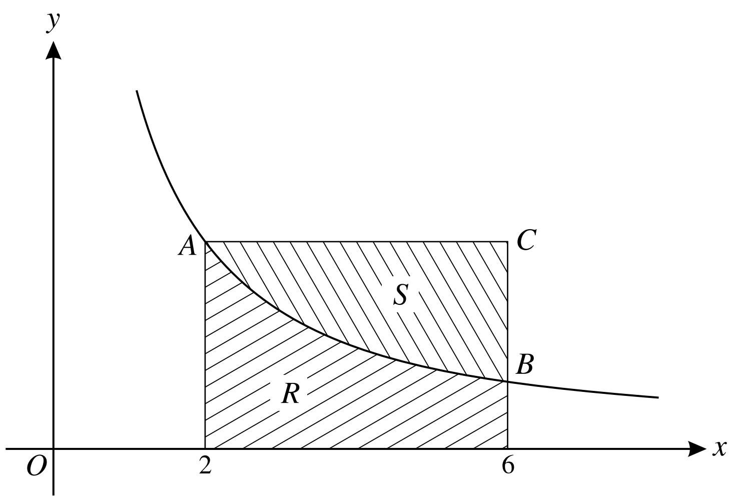
(i) The curve $y = \ln x$ is transformed by R followed by T. Find the equation of the resulting curve. **[2]**

(ii) Find, in terms of S and T, a sequence of transformations that transforms the curve $y = x^3$ to the curve $y = \left(\frac{1}{9}x - 4\right)^3$. You should make clear the order of the transformations. **[2]**

3 **(i)** Express the equation $\operatorname{cosec}\theta(3\cos 2\theta + 7) + 11 = 0$ in the form $a\sin^2\theta + b\sin\theta + c = 0$, where a, b and c are constants. **[3]**

(ii) Hence solve, for $-180^\circ < \theta < 180^\circ$, the equation $\operatorname{cosec}\theta(3\cos 2\theta + 7) + 11 = 0$. **[3]**

4



The diagram shows part of the curve $y = \frac{k}{x}$, where k is a positive constant. The points A and B on the curve have x-coordinates 2 and 6 respectively. Lines through A and B parallel to the axes as shown meet at the point C. The region R is bounded by the curve and the lines $x = 2$, $x = 6$ and $y = 0$. The region S is bounded by the curve and the lines AC and BC. It is given that the area of the region R is $\ln 81$.

(i) Show that $k = 4$. **[3]**

(ii) Find the exact volume of the solid produced when the region S is rotated completely about the x-axis. **[4]**

5 **(i)** Solve the inequality $|2x + 1| \leqslant |x - 3|$. **[5]**

(ii) Given that x satisfies the inequality $|2x + 1| \leqslant |x - 3|$, find the greatest possible value of $|x + 2|$. **[2]**

6 **(i)** Show by calculation that the equation

$$\tan^2 x - x - 2 = 0,$$

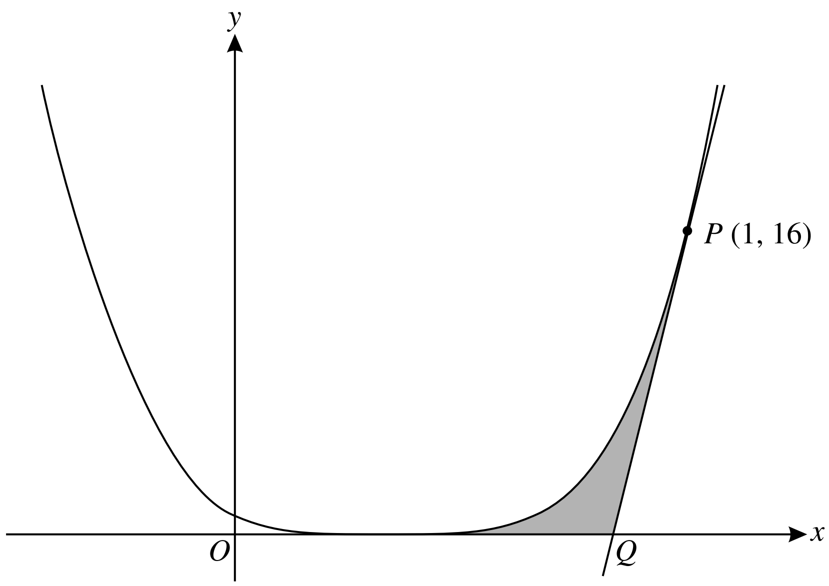
where x is measured in radians, has a root between 1.0 and 1.1. **[3]**

(ii) Use the iteration formula $x_{n+1} = \tan^{-1}\sqrt{2 + x_n}$ with a suitable starting value to find this root correct to 5 decimal places. You should show the outcome of each step of the process. **[4]**

(iii) Deduce a root of the equation

$$\sec^2 2x - 2x - 3 = 0.$$ **[3]**

7



The diagram shows the curve with equation $y = (3x - 1)^4$. The point P on the curve has coordinates (1, 16) and the tangent to the curve at P meets the x-axis at the point Q. The shaded region is bounded by PQ, the x-axis and that part of the curve for which $\frac{1}{3} \leqslant x \leqslant 1$. Find the exact area of this shaded region. **[10]**

8 **(i)** Express $3\cos x + 3\sin x$ in the form $R\cos(x - \alpha)$, where $R > 0$ and $0 < \alpha < \frac{1}{2}\pi$. **[3]**

(ii) The expression $\mathrm{T}(x)$ is defined by $\mathrm{T}(x) = \dfrac{8}{3\cos x + 3\sin x}$.

(a) Determine a value of x for which $\mathrm{T}(x)$ is not defined. **[2]**

(b) Find the smallest positive value of x satisfying $\mathrm{T}(3x) = \frac{8}{9}\sqrt{6}$, giving your answer in an exact form. **[4]**

[Question 9 is printed overleaf.]

9 The functions f and g are defined for all real values of x by

$$f(x) = 4x^2 - 12x \qquad \text{and} \qquad g(x) = ax + b,$$

where a and b are non-zero constants.

(i) Find the range of f. **[3]**

(ii) Explain why the function f has no inverse. **[2]**

(iii) Given that $g^{-1}(x) = g(x)$ for all values of x, show that $a = -1$. **[4]**

(iv) Given further that $gf(x) < 5$ for all values of x, find the set of possible values of b. **[4]**

1	**(i)**	Attempt use of product rule	M1	producing … + … form
		Obtain $3x^2e^{2x}+2x^3e^{2x}$	A1 **2**	or equiv
	(ii)	Attempt use of chain rule to produce $\dfrac{kx}{3+2x^2}$ form	M1	any constant k
		Obtain $\dfrac{4x}{3+2x^2}$	A1 **2**	
	(iii)	Attempt use of quotient rule	M1	or equiv; condone u/v confusions
		Obtain $\dfrac{2x+1-2x}{(2x+1)^2}$ or $(2x+1)^{-1}-2x(2x+1)^{-2}$	A1 **2**	or (unsimplified) equiv

[If $...+c$ included in all three parts and all three parts otherwise correct, award M1A1, M1A1, M1A0; otherwise ignore any inclusion of $...+c$.]

6

2	**(i)**	Obtain one of $\pm\ln(\pm x\pm 4)$	M1	
		Obtain correct equation $y=-\ln(x-4)$	A1 **2**	or equiv; condone use of modulus signs instead of brackets
	(ii)	State, in any order, S, S and T	M1	or equiv such as S^2, T or 2S, T
		State T, then S, then S	A1 **2**	or equiv (note that S, S, T^9 and S, T^3, S are alternative correct answers)

4

3	**(i)**	Use $\operatorname{cosec}\theta=\dfrac{1}{\sin\theta}$	B1	
		Attempt to express equation in terms of $\sin\theta$	M1	using $\cos 2\theta=\pm 1\pm 2\sin^2\theta$ or equiv
		Obtain or clearly imply $6\sin^2\theta-11\sin\theta-10=0$	A1 **3**	or $-6\sin^2\theta+11\sin\theta+10=0$
	(ii)	Attempt solution to obtain at least one value of $\sin\theta$	M1	should be $s=-\frac{2}{3},\frac{5}{2}$
		Obtain –41.8	A1	allow –42 or greater accuracy
		Obtain –138	A1 **3**	or greater accuracy; and no others between –180 and 180
		[Answer(s) only: award 0 out of 3.]		

6

4	(i)	Either: Integrate to obtain $k\ln x$	B1		
		Use at least one relevant logarithm property	M1		
		Obtain $k\ln 3 = \ln 81$ and hence $k = 4$	A1	**3**	AG; accurate work required
		Or 1: (where solution involves no use of a logarithm property)			
		Integrate to obtain $k\ln x$	B1		
		Obtain correct explicit expression for k and conclude $k = 4$ with no error seen	B2	**3**	AG; e.g. $k = \frac{\ln 81}{\ln 6 - \ln 2} = 4$
		Or 2: (where solution involves verification of result by initial substitution of 4 for k)			
		Integrate to obtain $4\ln x$	B1		
		Use at least one relevant logarithm property	M1		
		Obtain $\ln 81$ legitimately with no error seen	A1	**3**	AG; accurate work required
	(ii)	State volume involves $\int \pi(\frac{4}{x})^2 \, dx$	B1		possibly implied
		Obtain integral of form $k_1 x^{-1}$	M1		any constant k_1 including π or not
		Use correct process for finding volume produced from S	M1		$\int (k_2 2^2 - k_3 y^2)\, dx$, including π or not with correct limits indicated; or equiv
		Obtain $16\pi - \frac{16}{3}\pi$ and hence $\frac{32}{3}\pi$	A1	**4**	or exact equiv
			7		

5	(i)	Attempt process for finding both critical values	M1		squaring both sides to obtain 3 terms on each side or considering 2 different linear eqns/inequalities
		Obtain –4	A1		
		Obtain $\frac{2}{3}$	A1		
		Attempt process for solving inequality	M1		table, sketch, …; needs two critical values; implied by plausible answer
		Obtain $-4 \le x \le \frac{2}{3}$	A1	**5**	with $\le$ and not $<$
	(ii)	Use correct process to find value of $\lvert x+2 \rvert$ using any value	M1		… whether part of answer to **(i)** or not
		Obtain $2\frac{2}{3}$ or $\frac{8}{3}$	A1	**2**	dependent on 5 marks awarded in part **(i)**
			7		

6	**(i)**	Attempt calculations involving 1.0 and 1.1	M1	using radians
		Obtain -0.57 and 0.76	A1	or values to 1 dp (rounded or truncated); or equivs (where eqn rearranged)
		Refer to sign change (or equiv for rearranged eqn)	A1 **3**	AG; following correct work only
	(ii)	Obtain correct first iterate	B1	using value x_1 such that $1.0 \le x_1 \le 1.1$
		Carry out iteration process	M1	obtaining at least 3 iterates in all so far
		Obtain at least 3 correct iterates	A1	showing at least 3 dp
		Obtain 1.05083	A1 **4**	answer required to exactly 5 d.p.
		[$1 \rightarrow 1.047198 \rightarrow 1.050571 \rightarrow 1.050809 \rightarrow 1.050826 \rightarrow 1.050827$; $1.05 \rightarrow 1.050769 \rightarrow 1.050823 \rightarrow 1.050827 \rightarrow 1.050827$; $1.1 \rightarrow 1.054268 \rightarrow 1.051070 \rightarrow 1.050844 \rightarrow 1.050829 \rightarrow 1.050827$]		
	(iii)	State or imply $\sec^2 2x = 1 + \tan^2 2x$	B1	
		Relate to earlier equation	M1	by halving or doubling answer to **(ii)** or carrying out equivalent iteration process
		Deduce $2x = 1.05083$ and hence 0.525	A1√ **3**	following their answer to **(ii)**; or greater accuracy
		[SC: Rearrange to obtain $x = \frac{1}{2}\cos^{-1}(2x+3)^{-\frac{1}{2}}$	B1	
		Use iterative process to obtain 0.525	B1 **2**	or greater accuracy]
			10	
7		Differentiate to obtain $k_1(3x-1)^3$	M1	any constant k_1
		Obtain correct $12(3x-1)^3$	A1	or (unsimplified) equiv
		Substitute 1 to obtain 96	A1	
		Attempt to find x-coordinate of Q	M1	using tangent with $y = 0$ or using gradient
		Obtain $\frac{5}{6}$	A1	or exact equiv
		Integrate to obtain $k_2(3x-1)^5$	M1	any constant k_2
		Obtain correct $\frac{1}{15}(3x-1)^5$	A1	or (unsimplified) equiv
		Use limits $\frac{1}{3}$ and 1 to obtain $\frac{32}{15}$	A1	
		Attempt to find shaded area by correct process	M1	integral – triangle or equiv
		Obtain ($\frac{32}{15} - \frac{1}{2} \times \frac{1}{6} \times 16$ and hence) $\frac{4}{5}$	A1	or equiv
			10	
8	**(i)**	Obtain $R = 3\sqrt{2}$ or $R = \sqrt{18}$ or $R = 4.24$	B1	or equiv
		Attempt to find value of α	M1	condone sin/cos muddles and degrees
		Obtain $\frac{1}{4}\pi$ or 0.785	A1 **3**	in radians now
	(ii) a	Equate $x - \alpha$ to $\frac{1}{2}\pi$ or attempt solution of $3\cos x + 3\sin x = 0$	M1	condone degrees here
		Obtain $\frac{3}{4}\pi$	A1 **2**	or $\ldots, -\frac{5}{4}\pi, -\frac{1}{4}\pi, \frac{7}{4}\pi, \ldots$; in radians now
	b	Attempt correct process to find value of $3x - \alpha$	*M1	with attempt at rearranging $\mathrm{T}(3x) = \frac{8}{9}\sqrt{6}$
		Obtain at least one correct exact value of $3x - \alpha$	A1	$\pm\frac{1}{6}\pi, \pm\frac{11}{6}\pi, \ldots$
		Attempt at least one positive value of x	M1	dep *M
		Obtain $\frac{1}{36}\pi$	A1 **4**	
			9	

9	(i)	Attempt to find x-coord of staty point or complete square	M1		
		Obtain $(\frac{3}{2}, -9)$ or $4(x-\frac{3}{2})^2 - 9$ or -9	A1		or equiv
		State $f(x) \geq -9$	A1	3	using any notation; with $\geq$
	(ii)	Make one correct (perhaps general) relevant statement	B1		not 1 –1 , f is many-one, … ; maybe implied if attempt is specific to this f
		Conclude with correct evidence related to this f	B1	2	AG; (more or less) correct sketch; correct relevant calculations, …
	(iii)	Either: Attempt to find expression for g^{-1}	*M1		or equiv
		Obtain $\frac{1}{a}(x-b)$	A1		or equiv
		Compare $\frac{1}{a}(x-b)$ and $ax+b$	M1		dep *M; by equating either coefficients of x or constant terms (or both); or substituting two non-zero values of x and solving eqns for a
		Obtain at least $-\frac{b}{a} = b$ and hence $a = -1$	A1	4	AG; necessary detail required; or equiv
		[SC1: first two steps as above, then substitute $a = -1$: max possible M1A1B1]			
		[SC2: substitute $a = -1$ at start: Attempt to find inverse M1 Obtain $-x+b$ and conclude A1 **2**]			
		Or: State or imply that $y = g^{-1}(x)$ is reflection of $y = g(x)$ in line $y = x$	B1		
		State that line unchanged by this reflection is perpendicular to $y = x$	M2		
		Conclude that a is –1	A1	4	
	(iv)	State or imply that $gf(x) = -(4x^2 - 12x) + b$	B1		
		Attempt use of discriminant or relate to range of f	M1		or equiv
		Obtain $64 + 16b < 0$ or $9 + b < 5$	A1		or equiv
		Obtain $b < -4$	A1	4	
				13	